Drawing with Spheres in Two-Dimensions

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CIRCLES AND SPHERES

If we construct a material circle, measure its radius and circumference, and see if the ratio of these two lengths is equal to π , what shall we have done? We shall have made an experiment on the properties of the matter with which we constructed this round thing, and of that which the measure used was made. [1]

—Henri Poincaré, *Science and Hypothesis*

Contemporary digital practice allows for an incredible facility in the both the generation and fabrication of objects. However, despite high degrees of numerical precision, geometric information is lost as is moves from the abstract (non-visual) world of geometry to the screen, paper or some other material form. The job of the architect, as described from Alberti onward, is to define the building through drawing so that it can be precisely constructed on site.[2] A process of approximation and loss along the way is less than ideal, but it is a central part of the digital culture of which architecture is a part.[3] It is also an intrinsic, but often times unmentioned, aspect of architectural method.[4] Even Henri Poincaré's ideal circle is manifested as only a "round thing" once it translated into physical space—leaving the designer's intent and the form of the object muddled with technologies of fabrication. This project questions ideas precision in architectural drawing through the study of approximation and loss as generative drawing strategies. The project is composed of a series of animations that and still images produced through the continuous approximation of both a simple and iconic geometry—the sphere.

The sphere, like the circle, is an ideal geometry that can only be approximated in material form. A key example of this from the history of architecture is in the drawing and building of hemispherical domes from stone blocks. A hemisphere is a double curved surface that cannot be unrolled without first being transferred to another geometry. One common technique for this, was the use the use of the cone, a developable surface, to unroll the surface of the sphere.[5] The process involves dividing the vertical section of the sphere into segments. A cone is then located that is tangent to the surface of sphere at each segment and whose vertex lies on ray that passes through the center of the sphere. If a hemisphere is divided into eight subdivisions, it will require the unrolling of eight unique cones for the outside of the dome and eight unique cones for the inside of the dome.[6] All sixteen cones will possess vertices that align along a single ray that also passes through the center of the sphere (Figure 2). The hemisphere is not generated as an object and then developed by the cones. Instead, the entire graphic description and process of generating the hemispherical dome is controlled by the geometric properties of another figure, the cone. The cone is a generative drawing devise in which approximation is used not simply to render a close relative of the ideal hemisphere, but rather to produce a new object whose properties are a composite of spherical and conical geometries.

PROCESS

This project begins with the sphere and uses its approximation in points and lines as a method for the generation of a series of two-dimensional animations rendered here as images. The trigonometric equation for the calculation of points on a sphere provided a simple method for approximating spherical geometry. When the calculation is performed iteratively it produces an ordered list of points that cover the surface of a sphere. Passing lines through these points will provide an approximation of the sphere as a polyline. If a point is generated every forty-five degrees of rotation, the sphere will be approximated with a set of eight-side polygons(Figure 3). The more points that are used the closer the approximation is to the surface of the ideal sphere.

The above operations were encapsulated into a program written in Python for the software environment Processing.py.[7] The program uses a set of animated two-dimensional descriptions of eight-interconnected spheres (Figure 1). As the program runs it changes the approximation of the spheres at intervals. Each time the approximation changes, the drawing changes both in terms of the geometry it describes and the prioritization of color between black and blue (Figures 4-5). There are four kinds of drawing processes within the animations. The first corresponds to the conventional top, front and side views of architectural and engineering drawing. The second is an oblique

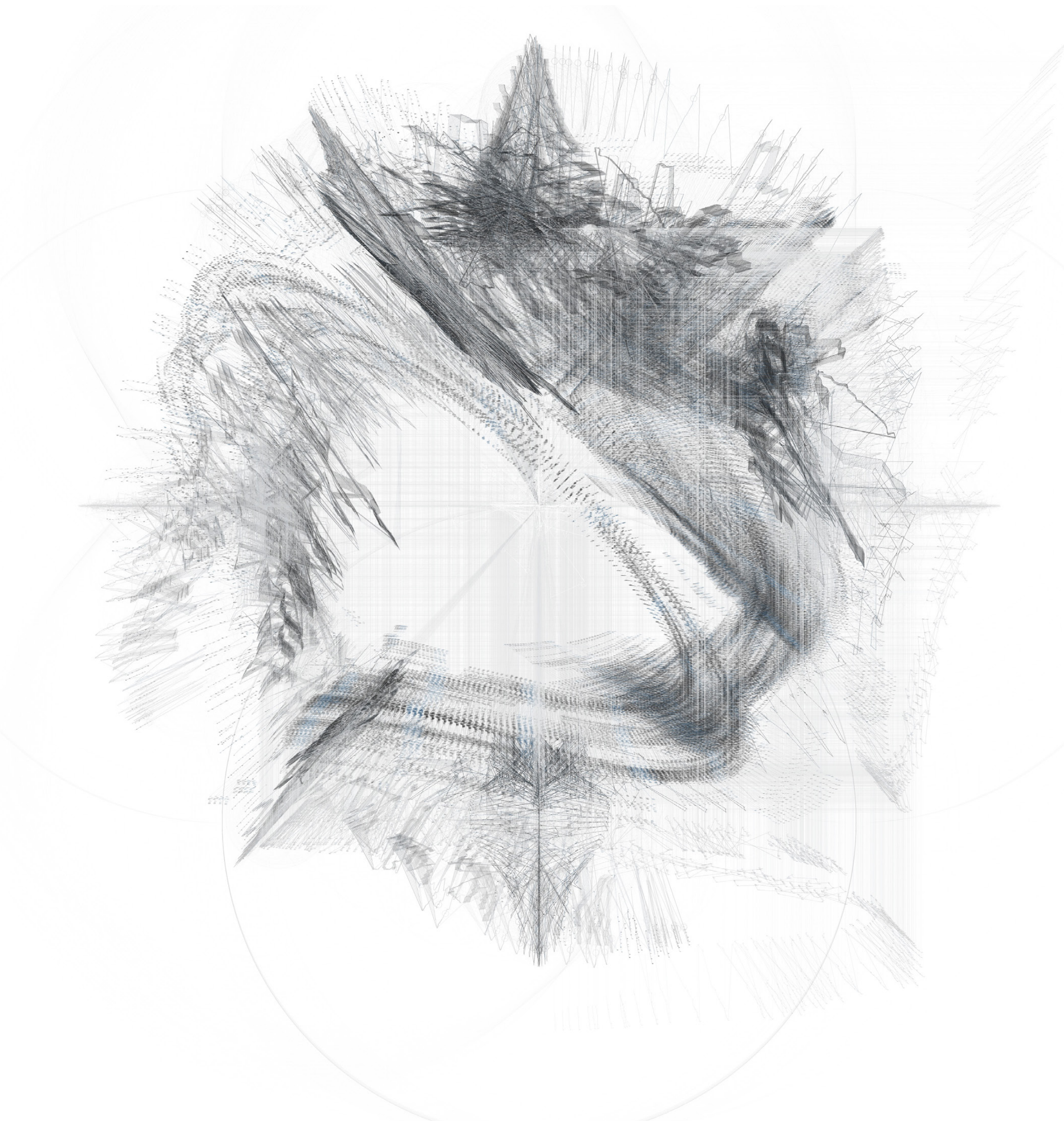

Figure 1.DWS2D Version 139. This is the last frame of the animation and contains all 18 drawings. Image by Author.

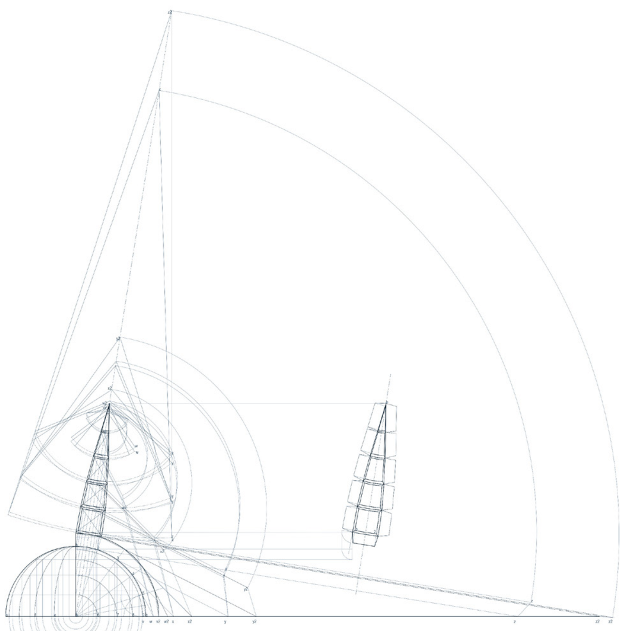

Figure 2. Hemispherical Dome, Stereotomic Projection. Image by Author

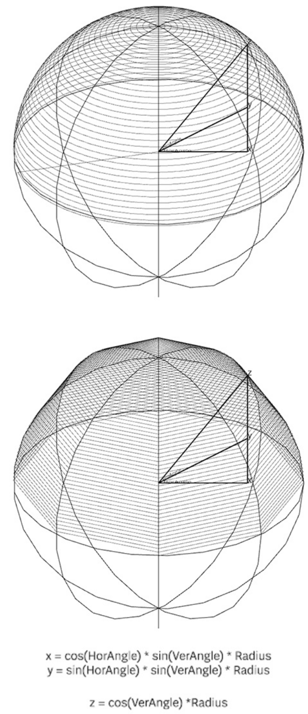

Figure 3. Spherical Coordinates Explained. Image by Author

process in which the “round thing” is displaced along an axis to simulate a three-dimensional view (Figure 6). The third is a drawing process that unrolls all the of the previously described drawings along the circumference of the largest sphere (Figures 7-8). The fourth is an orthographic projection of the unrolled drawings (Figures 9-10). There are a total of eighteen different drawings within an animation, each of which correspond to one of the four previously mentioned processes. Every drawing process tracks the motion of eight interconnected spheres that are continuously being approximated by variable number of points and lines.

Approximation is fundamental to both describing and representing curves. Stereotomy, the discipline of drawing the templates for the cutting of stones for stone vaults, used the ruled surfaced to approximate and unroll the doubly curved surfaces of spherical and ellipsoidal domes. Contemporary text books on computational geometry suggest the use of polylines to estimate the intersection of polynomial curves because it is less computationally expensive.[8] Even when we consider the screen-based representations of the circle we find approximations in the form of lines and NURBS curves.[9] In all cases, the approximation operates as a stand in for the ideal geometry allowing it be seen, fabricated or calculated with. This project inverts this hierarchy of representation, by positioning approximation as a generative drawing device in lieu of a representation. The animations are precise miscalculations of ideal geometries. There are no spheres or circles, only round things rendered in the material of which “the measure was made”—points and lines.

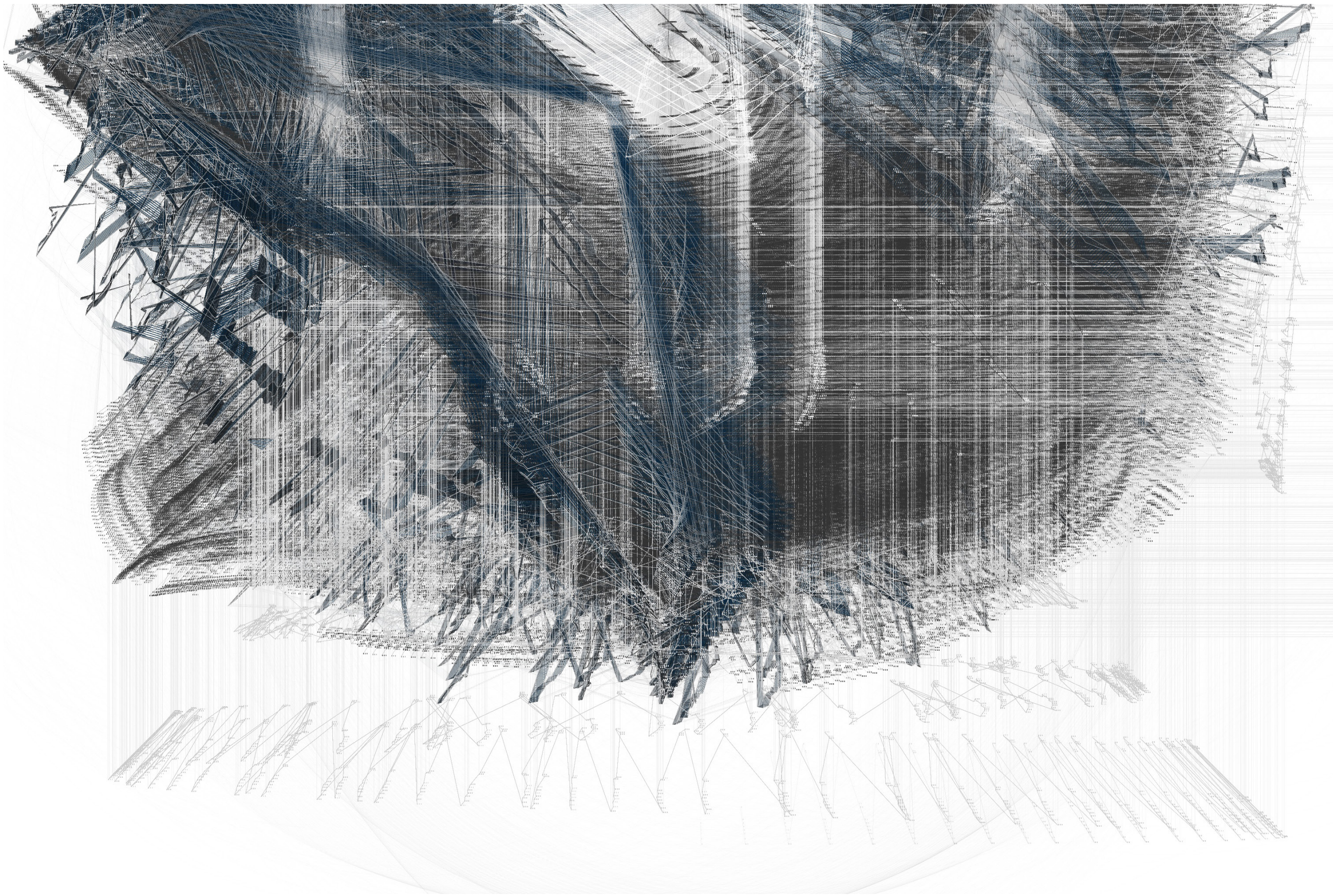

Figure 4. DWS2D version 122, Detail. Color shifts along a linear gradient between black and blue. Image by Author

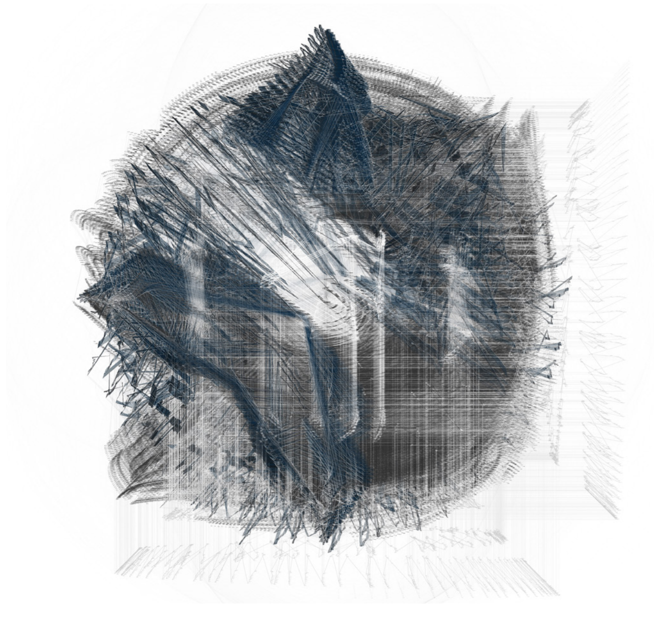

Figure 5. DWS2D version 122. Image by Author.

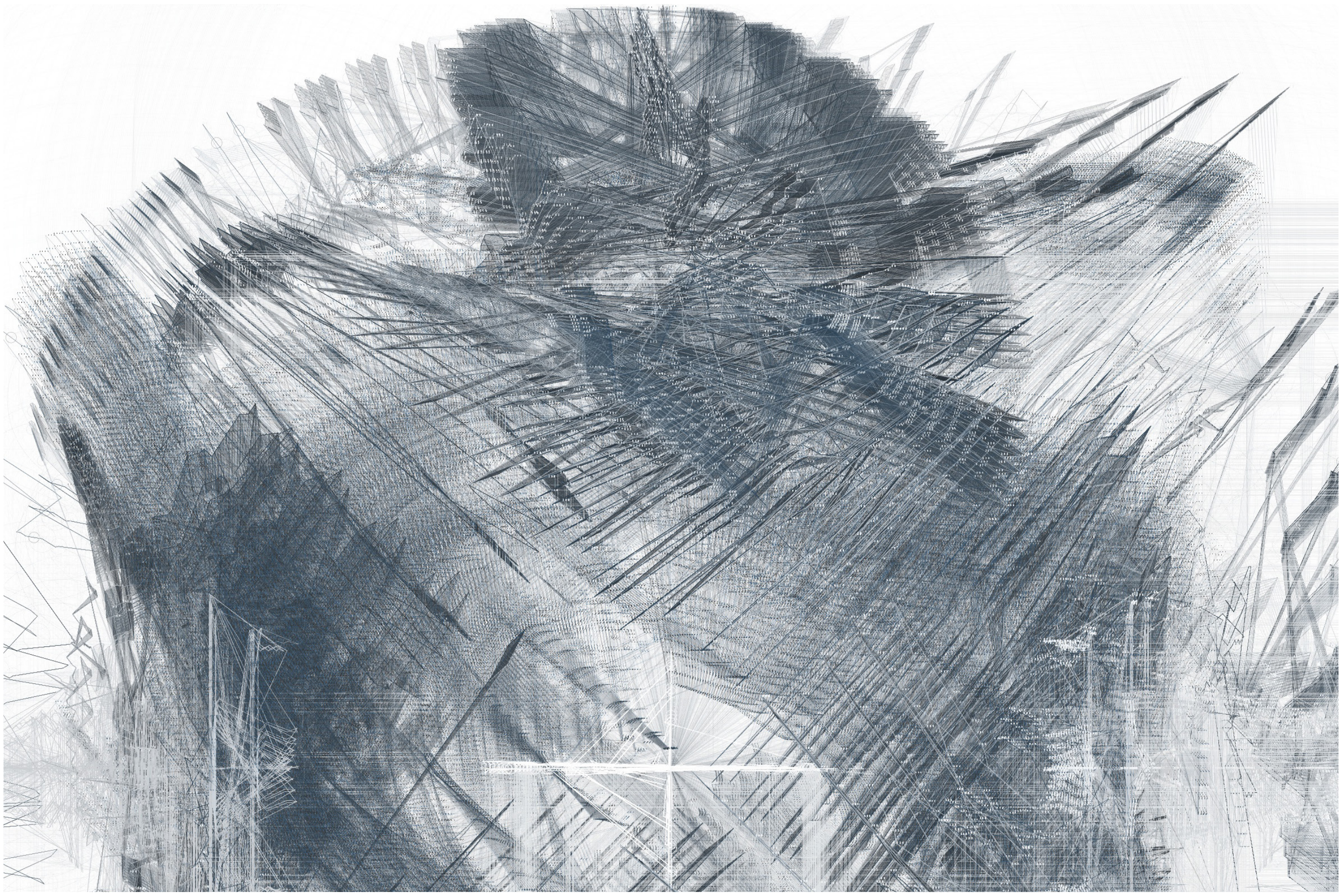

Figure 6. DWS2D version 119, Detail. Objects are given thickness with two approximate spheres and hatched. Image by Author

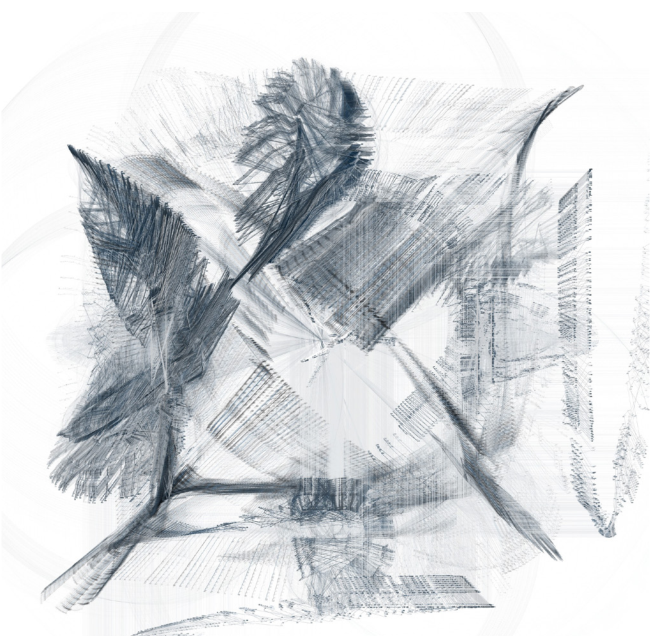

Figure 7. DWS2D version 90. Image by Author

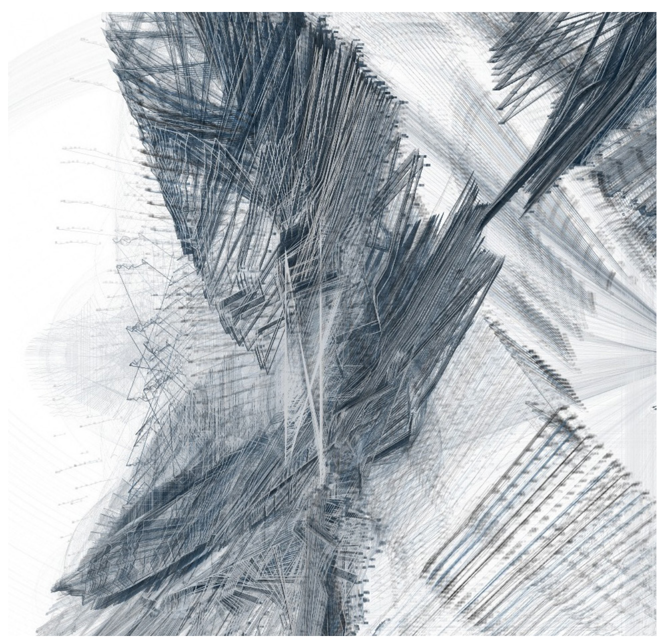

Figure 8. DWS2D version 90. Oblique and Unrollled view. Image by Author

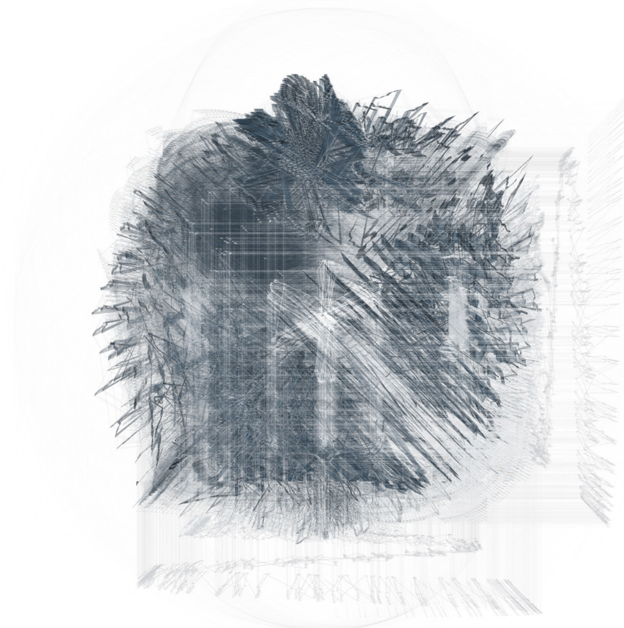

Figure 9. DWS2D version 114. Image by Author

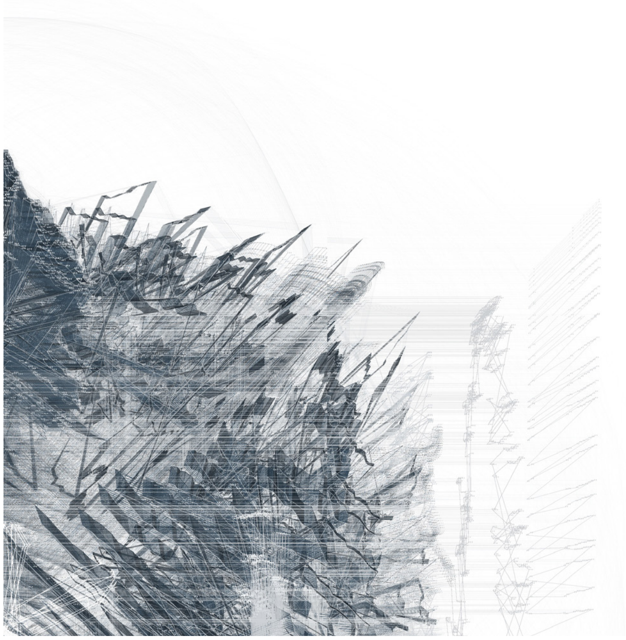

Figure 10. DWSD version 114, Detail. Hatched and unrolled sphere with orthographic projection (right side). Image by Author.

ENDNOTES

1. Henri Poincare, ***Science and Hypothesis*** (New York: The Science Press, 1913).
2. 2 Leon Battista Alberti. ***The Art of Buildng in Ten Books***. Translated by Joseph Rykwert, Neal Leach, and Robert Tavenor. Cambridge: MIT Press, 1988.
3. Lev Manovich. “Paradoxes of Digital Photography.” In ***Photography after Photography: Memory and Representation in the Digital Age***. Germany: G+B Arts, 1995.
4. “What comes out is not always what goes in. Architecture has nevertheless been thought of as an attemptat maximum preservation in which both meaning and likeness are transported from idea through drawingto building with a minimum of loss. This is the doctrine of essentialism… ” in Robin Evans. “Translations from Drawing to Buildings.” In ***Translations from Drawing to Building***, 153–89. Cambridge: MIT Press, 1997.
5. Joseph Gwilt, ***The Encylopedia of Architecture*** (London: Longmans, 1867).
6. Guarino Guarini, ***Architettura Civile*** (Turin: G. Mairesse, 1737).
7. Casey Reas and Ben Fry, ***Processing: A Programming Handbook for Visual Designers and Artists***, 2nd ed. (Cambridge, Mass.: MIT Press, 2014).
8. David Eberly and Philip Schnieder, ***Geometric Tools for Computer Graphics*** (San Francisco: Elsevier Science, 2003).
9. In the case of the NURBS circle, the eight points positioned around the perimeter of a square have alternating weights. The weights of the four control points that are tangent to the edges of the square are equal to 1.0. The weights that are connected to the control points at the four corners of the square are equal the cosine of $\pi/4$. The cosine of $\pi/4$ is equal to both the “x” and “y” coordinates of a point on the unit circle at forty-five degrees. See Rajaa Issa, ***Essential Mathematics for Computational Design***, 4th ed. (Robert McNeel & Associates, 2019).

PROJECT